Whitepaper

Fostering Student Engagement in the Mathematical Practices

Using Instructional Routines That Develop Productive Habits for Success

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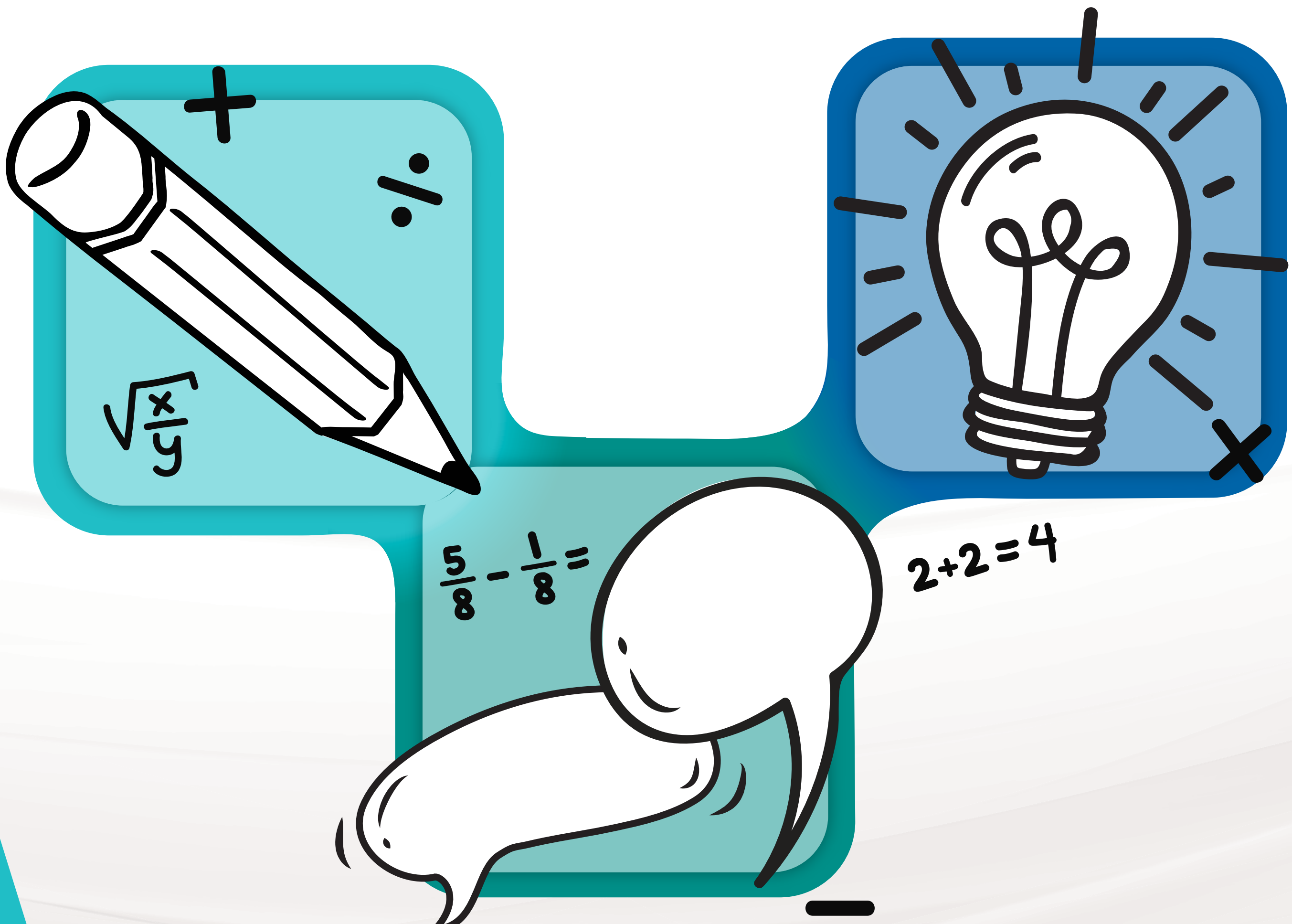

i-Ready Classroom Mathematics

About the Author

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Mark is a professor of education at California State University, Fullerton and a highly regarded scholar and teacher of Grades K–12 mathematics education. With more than 40 publications to his credit, Mark has served on the National Council of Teachers of Mathematics (NCTM) Board of Directors and Executive Committee and recently published a book about creating productive mathematics learning environments for elementary students. He is best known for his collaborative work with educators in developing strategies that help students understand concepts, supporting instruction, and addressing issues of equity in mathematics education.

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Introduction

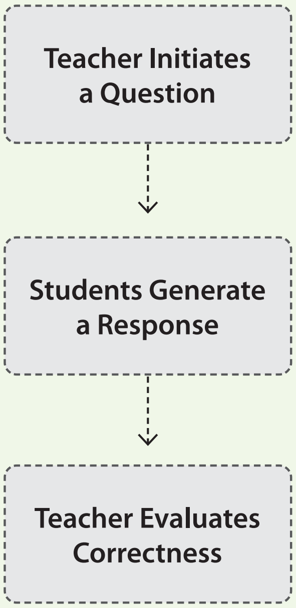
Routines are an essential part of our everyday lives. Just think for a moment about how routines associated with common activities, such as getting ready for work in the morning, provide a flow that governs the actions taking place. Your morning routine, including when you brush your teeth, shower, dress, and eat breakfast, is likely done in the same order and in the same way each day. Your drive to work likely follows the same path, and you likely have a routine for starting your day at work. School is another place that thrives on routines. How else would a small number of adults survive 180 days among hundreds of energetic young people? What is at issue in what follows is not whether we must have routines—that is a given. The issue, rather, is about the nature of these routines and the interactions they are intended to nurture.

For many educators, myself included, the typical US mathematics lesson in much of the 20th century (and into the 21st) consisted of the teacher showing how to solve a new type of problem while students took notes, followed by students working, usually individually, on completing a set of similarly structured problems by mimicking the teacher's steps (or those from the textbook) while the teacher checked their work (Ellis, 2007; Stodolsky, 1988). In this way, learning mathematics involved some variation of a ubiquitous standard instructional routine as shown in Figure 1.

This routine is often referred to in academic literature as "initiation-response-evaluation" or just IRE (Mehan, 1979). While the IRE or "I do, we do, you do" routine might produce short-term knowledge of procedural skills, research over many decades has shown it to be ineffective for most students in developing conceptual understanding, long-term procedural fluency, and mathematical reasoning skills (National Research Council, 2001).

What's more, the widespread use of IRE has exacerbated the common preconception that mathematics ability is found only in a small proportion of the population, leaving many students (and adults) with the mistaken belief that they are not good at mathematics (National Research Council, 2001). As Susan Stodolsky described in her 1988 book based on observations of dozens of elementary mathematics lessons across the United States, math instruction places all but the exceptional student in a position of almost total dependence on the teacher for progress through a course. In essence, the traditional math classes contain only one route to learning: teacher presentation of concepts followed by independent student practice.

Historically Common Initiation-Response-Evaluation (IRE) Instructional Routine



The widespread use of IRE has exacerbated the common preconception that mathematics ability is found in only a small proportion of the population, leaving many students (and adults) with the mistaken belief that they are not good at mathematics.

Figure 1: *IRE Instructional Routine*

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Within this behaviorism-based routine, students rarely had opportunities to generate original thinking and were expected to follow the mathematical authority of the teacher or textbook.

However, current mathematics standards, designed to help students learn the concepts behind the calculations, place an emphasis on students' regular use of mathematical practices (e.g., the Standards for Mathematical Practice in the Common Core State Standards for Mathematics) to develop deep understanding of mathematical ideas and relationships. These standards are predicated on the beliefs that a) mathematics makes sense and b) every student has the potential to engage in mathematical reasoning.

This shift in expectations for mathematics learning requires a classroom environment where productive problem solving is an everyday part of learning mathematics, and students are supported in developing understanding of important mathematical concepts, procedures, and relationships (Yeh, Ellis, & Hurtado, 2016). The instructional routines of the mathematics classroom—the rules governing interactions and activities during mathematics lessons—must likewise shift.

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Fortunately, there is a long line of research, originating in the 1950s and continuing into the 21st century, that has consistently identified the importance of **students' focused cognitive engagement together with strategic teacher guidance** for promoting deep learning in mathematics (Honomitchl & Chen, 2012; Loibl et al., 2016; Mayer, 2004; National Research Council, 2001). While there is no one method that must be followed, it is increasingly clear that, "Students need enough freedom to become cognitively active in the process of sense making, and students need enough guidance so their cognitive activity results in the construction of useful knowledge" (Mayer, 2004). This general sequence of student active cognition followed by teacher-guided instruction has been referred to as "problem solving followed by instruction" or PS-I (Loibl et al., 2016).

The PS-I sequence leads to learning that is deeper and longer lasting than either initial direct teacher instruction or open, unguided student exploration (DeCaro & Rittle-Johnson, 2012; Kapur, 2015; Loibl et al., 2016). Within this perspective, problem solving:

> *involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and related patterns of inference that resolve some tension or ambiguity that prompted the original problem-solving activity. (Lester & Kehle, 2003)*

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This acknowledges that the reasoning, revising, and connecting of ideas that occurs both during and after students' productive engagement in problem solving is far more valuable in furthering their understanding of mathematics than simply obtaining correct answers (Lambdin, 2003; Lester & Cai, 2016) and allows us to recognize **learning through problem solving** as a strategy that promotes making sense of new concepts or relationships within mathematics.

One way to guide students to learn through problem solving is to use the **Try–Discuss–Connect** (TDC) instructional routine. Research suggests that the regular and productive implementation of TDC can significantly increase student success with deep learning in mathematics within the PS-I structure. One possible model (among many variations) for using TDC is shown in Figure 2.

When using TDC, the learning sequence should begin with something that intentionally triggers students' prior knowledge and generates curiosity about a problem[1] that students can understand but cannot quickly resolve. The problem may be found within prepared curriculum materials, though it is also possible for teachers to create problems themselves (and even better to do so collaboratively with students). The characteristics of a productive problem include a) requiring knowledge just beyond what students possess, b) allowing multiple solution pathways without an immediately obvious solution, and c) compelling student thinking about one or two specific mathematical features (e.g., concepts, relationships, properties, representations).

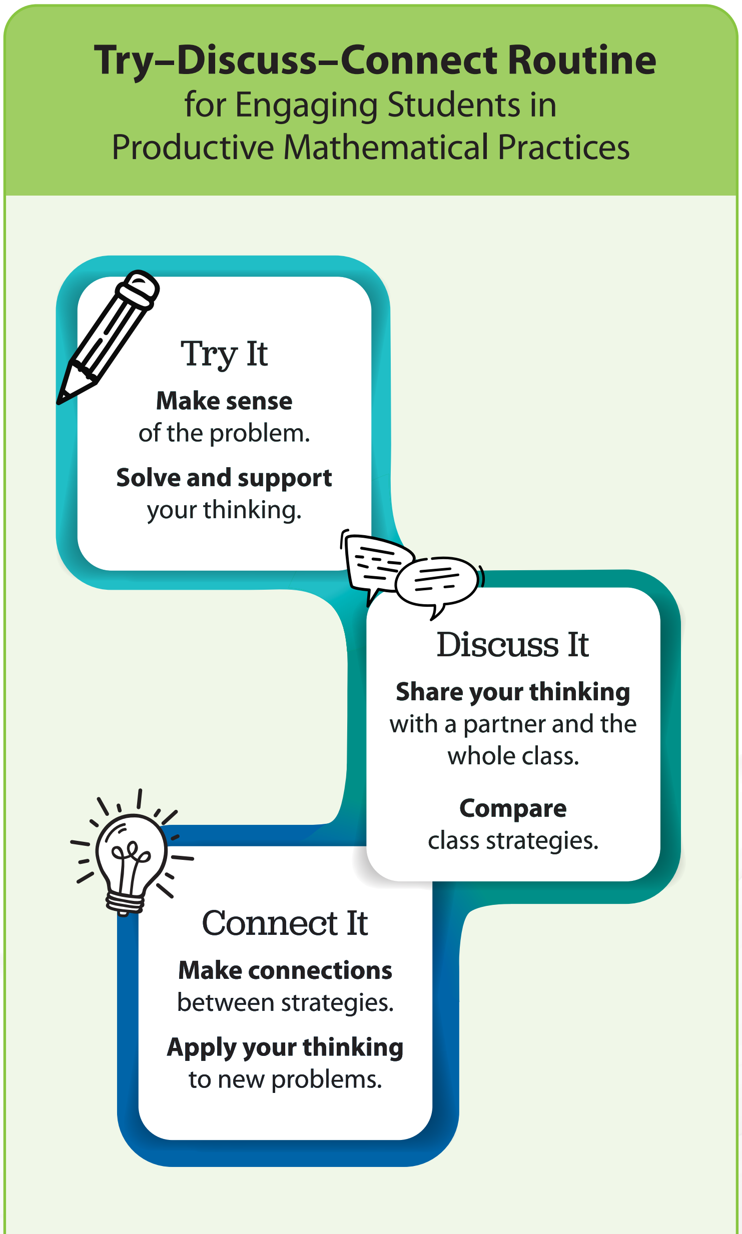

Figure 2: *A Model for Using the TDC Routine*

[1] The term "problem" as used here is intended to be taken as synonymous with task and scenario, two terms also used within mathematics education to denote items that are used as vehicles for learning and not merely practice.

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Understanding the Try–Discuss–Connect Routine: How It Works

Within the Try–Discuss–Connect routine, students do most of the thinking and talking, and the teacher serves as a facilitator. Let's examine the role of students and teacher within each of the phases of TDC.

Try It

Make Sense of the Problem

Activating student thinking takes place in two parts: The first part, which is often overlooked, is helping students to make sense of the problem or task they are about to attempt. In this phase, students and teacher read the problem multiple times to ensure students understand the context, know what they are trying to do or find, and identify key information or vocabulary. This part of the Try It phase is extremely important, especially for students who are emerging readers or are not native English speakers. It gives all students the opportunity to better understand the problem or task at hand, and it also reinforces the habit of making sense of the problem before solving it, as highlighted in Standard for Mathematical Practice (SMP) 1. When this step is omitted, students seldom read the problem for understanding, and teachers will often hear, "I don't know how to do this," or "I don't know where to start." Spending enough time on this step is critical to helping students develop effective problem-solving habits.

Solve and Support Your Thinking

The second part of the Try It phase involves giving students time to develop their own strategies and choose their own ways to model a given problem with mathematics (SMP 4). It is important to allow time for students to individually generate ideas about the problem, even if they are not able to reach a solution. Encourage them to try more than one strategy, including the use of visual models. Simply writing an answer is not sufficient. Students need to be able to represent and explain their strategy concretely, pictorially, representationally, or symbolically. As part of this, it is important for students to have manipulatives or mathematical tools accessible to them at all times, so they can choose appropriate tools to help them in solving the problem (SMP 5).

Allowing students time to explore each problem or task is critical to developing deeper understanding of the mathematical concepts and relationships. "If … the instruction does not build on student solutions, it seems unlikely that students [will] notice and elaborate on the deep features on their own" (Loibl et al., 2016).

For a quick summary of the Try It phase, see page 12.

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Discuss It

Discuss Strategies with a Partner or Small Group

Once students have had time to gather thoughts about the problem, they are ready to discuss their ideas with one or more peers. Each teacher should determine the optimal collaborative structure for sharing, whether pairs, triads, or groups of four.

Factors to keep in mind are the . . .

a) **Complexity** of the problem

b) **Prior knowledge** of the students

c) **Social relations** among students

While students work collaboratively to discuss solution strategies, **the teacher must resist the urge to step in, correct, and redirect, as these actions interrupt students' thinking**. During this phase, even incorrect or incomplete thinking is better than no thinking. Instead, the teacher should pose questions and prompts that elicit thinking as well as call attention to and address any issues (e.g., when one student is ignored by their partner or group or is not actively participating in the discussion). This portion of the Discuss It phase gives students opportunities to realize gaps and limitations in their thinking and to practice explaining their thinking in the relative safety of their small group. This is preparing their minds to be open to whole class discussion and instruction in the upcoming parts of the problem-solving discussion.

To promote peer conversations, you may want to provide students with sentence starters or questions they might ask one another. These help students to understand each other's thinking and work toward a common agreement about the solution (though they may not arrive at the solution during this time). Some examples include:

- *“The strategy I used was . . . because . . .”*
- *“I found the answer by . . .”*
- *“I got stuck here because . . .”*
- *“How did you get started?”*
- *“Can you explain more about what you did here?”*
- *“Do you agree with me? Why or why not?”*
- *“I am not sure about ____________. What do you think?”*
- *“Why did you __________?”*
- *“What other strategies or representations can we use?”*
- *“How do we know our result makes sense?”*
- *“What did we notice about this problem that was new or different?”*

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An alternative use of the Discuss It phase involves having students examine two contrasting solution strategies given by the teacher.

These would be specifically selected to call attention to mathematical features related to the learning goals. Loibl, Roll, and Rummel found this approach to be equally effective to having students share their own work. Students examining contrasting cases also begin to identify gaps in their knowledge and are thus more prepared for the whole class discussion and instruction that follows.

The key element in making this effective is that students have time in advance to develop initial ideas about the problem before examining those of others, whether peers' or predetermined cases.

During the peer conversations, the teacher is continuing to walk around the room, taking notes about which student strategies to select and in which sequence, for the whole conversation later in the Discuss It phase. In doing so, the teacher should keep in mind the mathematical goal(s) of the lesson and look for strategies that:

- Call attention to important features of the problem and/or key mathematical connections being developed
- Illustrate common misconceptions
- Reflect deeper understandings (even when not fully formed)

For example, if a number of students have used a strategy that reflects a misconception (e.g., adding denominators when adding two fractions), it may be good to start the class discussion by calling attention to this as a means to spark discussion. It is important to have students—rather than the teacher—discuss why an incorrect strategy may be reasonable (validating there is some mathematical meaning behind it) as well as whether they agree or disagree and why. This keeps students engaged in the thinking and conversation focusing on the mathematical sense making. The teacher should say something like, "I have seen this strategy before and wonder who can share why someone might think it is true?" Language such as this reinforces a growth mindset toward learning, and even incomplete strategies can be used to further our understanding. Along these lines, it's important that students learn to respectfully critique the strategies that are shared and not the person doing the sharing.

As you begin to make a list of strategies you would like shared, it is helpful to let students know in advance which of their strategies you would like them to explain. It is also helpful to the teacher as they prepare for the classroom conversation to keep track of the students who will share and which strategies the students will be sharing. See Dr. Gladis Kersaint's whitepaper *Selecting and Sequencing Student Solutions* (2017) for more information on the selecting and sequencing process.

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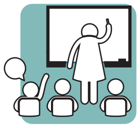
Discuss Strategies with the Whole Class

After students have generated strategies and had time to share with others and refine their thinking, their minds are ready for additional input and more formal guidance. During the whole class discussion, the teacher must orchestrate whole class discourse that uses selected student strategies to draw attention to important mathematical relationships and concepts, so-called "deep features" (Loibl et al., 2016).

. . . the teacher must orchestrate whole class discourse that uses selected student strategies *to draw attention to important mathematical relationships and concepts.*

During the whole class discussion, teachers facilitate the conversation, with students doing most of the explaining, critiquing, and restating (SMP 3). Often teachers have students come up and explain their strategy to the class. While this can have positive benefits in building student presentation skills, it is good to mix it up, too. Other productive approaches for whole class sharing include:

Have other students record for the class as the selected student(s) describes their strategy. This requires students to use more precise language and/or explanations of their strategies (SMP 6) and encourages greater listening between other students. Have the rest of the class signal thumbs up or thumbs down to show whether the recorder(s) is accurately representing what is being said by the student(s) sharing their strategy. This encourages all students to listen more carefully and helps them further develop their listening skills.



When the teacher is the recorder, be sure to write exactly what the student says. So for example, if a student says, "Draw a box," draw a three-dimensional rectangular prism rather than the two-dimensional rectangle you know they meant. This will help students attend to precision with their explanations and vocabulary (SMP 6).

Ask the selected student(s) to describe some of the steps of their strategies, and then ask one or more students to restate what the student(s) said or predict the next step of the solution strategy. Only when another student is restating the strategy does the teacher begin to visually record the process. This encourages students to listen to one another, rather than waiting for the teacher to restate or write what was said. It also increases listening skills and provides the teacher with formative assessment data about more of the students in the class.

There are also sentence starters teachers can share with students to prompt them to evaluate and politely critique student solutions during the whole class conversation (SMP 3). Note, again, how these focus attention on the student's strategy as the object of discussion.

> "*I agree with ______'s strategy because _________.*"

> "*I disagree with ______'s strategy because _________.*"

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Connect It

Make Connections between Strategies

It is not enough to have students discuss multiple strategies. In order to support deep conceptual understanding and to help students move from less sophisticated strategies to more sophisticated strategies, they must be supported in looking for and making sense of the mathematical connections within and among the strategies. Looking at the strategies for similarities is a key component of the classroom conversation. Students must also be able to demonstrate that they understand and can explain other strategies. During this phase the teacher may more directly step in to focus student thinking on one specific mathematical goal or relationship at a time. "The comparison of student solutions should explain one deep feature at a time, leading to a meaningful understanding of these features" (Loibl et al., 2016).

The process for making connections between strategies may involve two or three parts:

Give students individual think time to identify similarities between the strategies that have been discussed. You may want to provide students with prompts, such as

> "*What is the same about the strategies?*"

> "*Why do you think the strategies are alike?*"

> "*Do you see any similar structures or patterns?*" (SMP 7)

You may want to **allow students to compare the similarities they noticed with a partner**, but if you are short on time, you may want to jump immediately to the whole class discussion.

Identifying differences between some representations can also be useful, particularly if these help focus student attention on key mathematical features.

As a class, have students identify the similarities and/or differences they noticed in the strategies that were discussed. Encourage other students to ask questions about the connections or ask students to explain their reasoning.

Over time, as students become familiar with making connections, they will independently—by habit!—begin to look for features of problems that provide insight into new concepts and relationships (DeCaro & Rittle-Johnson, 2012). Give students time to test out, revise/reorganize, and demonstrate their understanding.

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Some teacher considerations to support the Connect It conversation include:

- Ask students to **compare the strategy they used** to those that were discussed.
- As students compare two of the strategies, **call attention to specific mathematical features relevant to the learning goals**. You may want to include a sentence starter to support this conversation, such as "When I compare _____'s strategy and _____'s strategy I notice . . . ."
- **With selected strategies, ask students to discuss which strategies . . .**
  –Are easiest to understand

  –Are most efficient

  –Can be used with other problems
- **Bring closure by asking students to write in their math notebook what they learned** about the topic of the lesson. Focus their attention on the key idea, relationship, procedure, or representation.
- **Provide students with questions and sentence starters** to encourage evaluation and reflection. These may be posted in the classroom or provided to students.

> "*Today I learned ______________. This will help me to ___________.*"

> "*I still have questions about ____________________.*"

> "*I wonder if/why ____________________.*"

> "*I would like to know more about ____________________.*"

- **Have students share their writing, and write a summary for the class** using language and representations borrowed from students' shared thoughts.

Apply Learning to New Problems and Situations

Once the classroom conversation and student journaling have been completed, have students apply what they have learned to new situations. When they do other problems or tasks, encourage students to show multiple ways they could solve the problem. In some cases, you may want students to try a new strategy or representation as one of the ways they show their solution. Ask students to make connections between the strategies they choose and explain the similarities between the different approaches.

Encourage students to ***show multiple ways*** *they could solve a problem.*

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Try–Discuss–Connect Quick Reference

Try It

What Students Do:	**Make sense of the problem or task as a class.**	**Solve the problem or task individually and explain their thinking.**
What Teachers Do:	• Ask multiple students to describe the context and identify words that may be confusing. • Ask multiple students clarifying questions to focus student understanding. • Guide students to greater precision in describing what the problem is asking.	• Make sure students have multiple concrete tools available to choose from to help them in solving the problem. • Observe students working on their strategies. • Begin to identify which two or three student strategies you want to discuss as a class.
Why?	• Clarify context and language that may be unfamiliar. • Support emerging readers and English Learners. • Ensure students understand what they are trying to do.	• Encourage multiple methods and representations. • Activate prior knowledge by working on a new problem. • Give students time to think about the problem. • Encourage student perseverance.

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Discuss It

What Students Do:	**Share their thinking with a partner.**	**Selected students share their thinking with the whole class.**
What Teachers Do:	• Listen to student conversations and identify which students you will have share their strategies and insights with the class. • Think about the sequence in which strategies might be shared, keeping the goals of the lesson in mind. • If there is a specific strategy you want students to consider (e.g., one highlighted in the textbook), look for a student who may have used that strategy or a strategy that can be used to build up to that strategy.	• Consider having students share an incorrect solution first, especially if a number of students made the same mistake. • Have students share their strategy with the class, sometimes with the student recording their process, sometimes with another student recording the process they describe. • Have students explain and critique their reasoning and that of others (SMP 3).
Why?	• Develop students' ability to explain their reasoning and critique the reasoning of others in a smaller setting. • Give students an opportunity to learn from the thinking of their peers.	• Develop students' ability to explain their reasoning and critique the reasoning of others. • If there is a specific strategy students did not generate but it is important to introduce, think about how to use student ideas to connect to it (e.g., "What strategies does this remind you of?", "Why do you think this strategy works?"). • Encourage a growth mindset by recognizing that every student has more learning to do, including the teacher, and we can learn from mistakes. Share a sense of surprise when a student shares a novel strategy or approach and promote a sense of curiosity when a strategy does not work, asking "Why is that?" • Students reason abstractly and quantitatively (SMP 2).

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Connect It

What Students Do:	**Make connections between solution strategies individually, in pairs, or as a class.**	**Apply multiple strategies to new problems individually or with a partner.**
What Teachers Do:	• Give students individual think time to identify similarities (and differences) among classroom strategies. • Have students identify connections among their strategies and those highlighted in the classroom conversation. • Guide students to explain the similarities and connections between various representations and strategies, particularly for those strategies that are standards-based or in the textbook.	• Encourage students to solve the problems in more than one way. • If students need help, ask questions that will get them thinking about a possible approach rather than directly telling them what to do. • As time allows, have students compare their strategies and answers with a partner and discuss similarities and differences. • Have students politely critique the reasoning or strategies of other students.
Why?	• Students make connections between what they know and new concepts, deepening their understanding. • Students reason abstractly and quantitatively (SMP 2). • Students build confidence and flexibility in solving problems as they see that there are many ways to approach a problem.	• Students practice what they have learned, choosing how to model new problems with mathematics (SMP 4). • Students independently choose appropriate tools when solving a problem (SMP 5).

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How the Try–Discuss–Connect Routine Works with a Sample Problem

Let's take a look at how TDC works by examining how you might approach introducing students to the idea of adding two two-digit numbers where the sum of the ones digits requires that students compose a 10 (CCSS Standard 1.NBT.C.4).

Connect to Prior Learning

Before beginning the new content, the teacher might begin the lesson by asking students to recall an earlier problem they worked on:

> *"One necklace has 26 beads. Another necklace has 32 beads. How many beads are there all together?"*

With their work on the problem displayed on chart paper next to the teacher (Figure 3), students are asked to remember how they approached this problem. Several students are asked to share their thinking process while the teacher visually connects these to the work on the chart paper and orally names the strategies (e.g., broke apart tens and ones; used part-part-whole). This allows students to recall their prior strategies for adding two-digit numbers and, although brief, helps prepare students for the thinking they will do about the new problem.

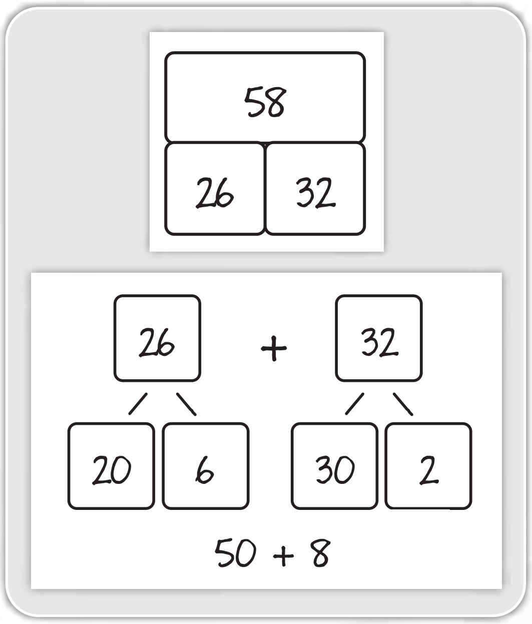

Figure 3: *Prior Work with Two-Digit Addition without Regrouping*

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Try It

Make Sense of the Problem

After connecting to prior learning, the teacher then introduces a new problem (Figure 4) about two groups of marbles. They first ask students what the problem is about, what they are trying to find, and then what information is important. Note that they do not yet ask students to solve the problem.

The teacher may ask questions like:

“What is a marble? How big is it? What is it made of?”

This is asked to ensure students understand the context and vocabulary.

“What are we trying to find?”

The teacher asks multiple students to share their ideas of what they are trying to find (NOT how they are going to find it). For example, students may say “the number of marbles,” “the total number of marbles,” or “how many marbles there are all together.” Encourage multiple students to share their thoughts, as this often leads to greater precision (SMP 6).

“What information is important?”

Here the teacher probes about information that students may need to pay particular attention to. In this more basic problem, the focus would be on the number of marbles in each group.

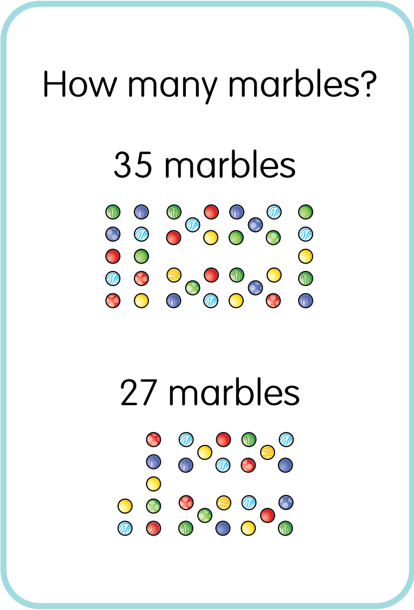

Figure 4: *New Problem That Requires Regrouping*

After making sense of the problem and noticing similarities to the bead problem, the teacher asks, *“Do you notice anything about this problem that is different than the bead problem (or others we have solved)?”* When one young scholar replies, *“The ones,”* the teacher probes further, *“What is different about the ones?”* The student continues, *“Seven and five make a teen number.”* The teacher has another student repeat what the first student said and then has students turn and talk to a partner, answering, *“Why is that different than other problems we have done?”* This is the critical moment when the lesson moves from revisiting prior knowledge to grappling with new ideas.

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Solve and Support Your Thinking

After guiding students to make sense of the problem, the teacher now gives students time to think about strategies to solve the problem. This is usually done individually at first, but it can also be done with a partner. Note that the teacher avoids giving hints about solution strategies and focuses only on clarifying the problem's information and overall goal as they walk around observing students' strategies.

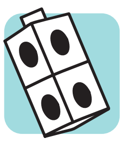
The following prompts (or variations of these that your own students come up with) might be posted in the classroom or in students' math notebooks as aids during the Try It phase:

> *Ask: " How is this related to what I already know? "*

> *Pose questions about the problem situation:*
> *" What might I want to know? "*

Make sure students ***always have access to tools and manipulatives***.

Other teaching practices to keep in mind during the Try It phase include:



Make sure students **always have access to tools and manipulatives** (e.g., counters, base-ten blocks, hundreds chart) so they are able to choose appropriate tools on their own (SMP 5).

Remind students to **model the problem using multiple strategies**, particularly if they have solved the problem one way (SMP 4).

Offer extensions of the problem for groups that might get to the mathematical insight earlier. For example, *"You did well to notice __________. Does that work in every case? Can you find an example of when it does not work? Or explain to me why it always works?"* (SMP 2).

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Discuss It

In the first part of the Discuss It phase of the routine, the teacher circulates around the room as pairs of students discuss their strategies for adding 35 marbles and 27 marbles to get the total number of marbles. She poses questions that get students to articulate their strategies to one another in more precise terms (SMP 6), asks students to reexamine their work so they might identify their own errors or incomplete strategies (SMP 2), and makes sure all students are participating equally. One key way to do this is to make sure a partner can explain the strategy another student used (SMP 3).

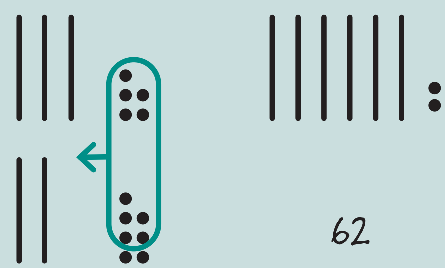
As students discuss their strategies, the teacher identifies students and strategies to use for the whole class discussion. They tell a few students the specific strategy they want them to share. In the second part of the Discuss It phase, selected students share their strategies with the class.

For example, the teacher may select and sequence students' strategies from the marble problem in this way:

1 Count on using a hundreds chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	81
91	92	93	94	95	96	97	98	99	100

2 Make a drawing to show base-ten blocks.



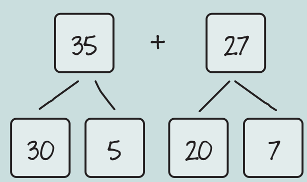
3 Add tens and then add ones.

$30 + 20 = 50$

$5 + 7 = 12$

$50 + 12 = 62$

4 Use a number bond to break apart the numbers into tens and ones.



$50 + 12 = 62$

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The following recommendations include best practices for teachers to use as they facilitate classroom conversations during the Discuss It phase:

Ask students to **share their thinking as explicitly as possible**.

Intentionally sequence selected student strategies to call attention to mathematical features important to the lesson.

Press for **precise use of already known vocabulary** and notation and **introduce new terms** and notation within the context of students' shared work.

Avoid giving evaluative responses such as, "That's right!"

Ask clarifying questions about students' thinking such as, "Tell us why you did this part" or "Tell us how this calculation is related to this representation."

For groups that seem to have an answer, **probe whether each member understands** how it was determined and why it makes sense.

As the teacher, don't restate students' comments or explanations. This tells students they don't have to listen to each other, because the teacher will restate it. Instead, **ask other students to restate what students say**.

Rather than the teacher identifying if a strategy or explanation is correct, **have the class show agreement or disagreement using hand signals like thumbs up or thumbs down**. Then have students who agree and those who disagree explain their thinking.

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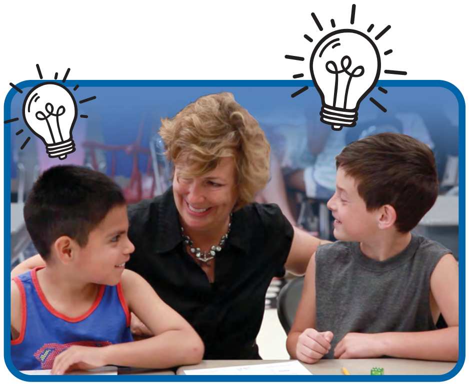

Connect It

In the first part of the Connect It phase, the teacher encourages students to see the similarities (and sometimes differences) among the strategies. In their responses, ask students to be precise in their use of language (and introduce new terms as needed in context). Looking at the strategies discussed with the class (Figure 4), the teacher may ask:

“How is the strategy using base-ten blocks similar to the strategy using the hundreds chart?”

“How is the strategy of adding tens and then adding ones like the strategy with the hundreds chart? How is it different?”

“How is the strategy of using the number bond like the strategy of using base-ten blocks?”

“How are all the strategies similar?”

At the end of the lesson, the teacher may ask questions like, *“What strategies did you see that will help you solve another problem like this?”* and *“Which of the strategies we discussed do you think is most efficient and why?”* as a way to help students reflect on their learning.

This reflection step helps focus student attention on generalizing a few strategies they may want to use with similar problems. Having them think about efficiency is a way to engage students in critical reasoning about the strategies to refine their understanding. That is the heart of it all, having students—even young first graders—actively engage in discussing mathematics, owning their learning, and realizing that there are many strategies that can be used to solve a problem. Some ways are more complex, some may take longer, but all can be used to approach the problem.

Students then apply what they have learned by practicing with new problems.

Conclusion

The use of a problem solving–based instructional routine that promotes true student and classroom discourse can help increase student understanding and retention of mathematics concepts. When students are given an opportunity to think about and attempt a problem, they have a greater ability to make connections and develop greater conceptual understanding. Through student-led discourse, students become active thinkers and learners. As students discuss their strategies, explain their reasoning, and evaluate and critique the reasoning of others, they internalize their learning and deepen their understanding. When supported by their teacher in noticing specific, important mathematical connections across multiple strategies, students develop a better sense of the coherence of mathematics and are better able to flexibly apply what they have learned to new situations. An added benefit of a routine like Try–Discuss–Connect is the way it naturally integrates the Standards for Mathematical Practice into instruction so these habits of mind become routine for students.

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